Heuristic and Exact Algorithms for the Cell Formation Problem

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Cell Formation Problem

Manufacturing process

Problem

Group machines and parts into production cells to minimize part movements and maximize cells loads

Cell Formation Problem

Example

	1	2	3	4	5	6	7
1	1	0	0	0	1	1	1
2	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0
4	1	1	1	1	0	0	0
5	0	1	0	1	1	1	0

King & Nakornchai 5x7 benchmark instance

	5	4	2	3	1	6	7
5	1	1	1	0	0	1	0
4	0	1	1	1	1	0	0
2	1	1	1	1	0	0	0
3	1	1	0	1	0	1	0
1	1	0	0	0	1	1	1

Solution

Cell #1 : M = {2,3,4,5}
P = {2,3,4,5}
Cell #2 : M = {1}
P = {1,6,7}

Approaches

- Clustering methods (ROC, ZODIAC, GRAPHICS)
- Graph methods (clique partitioning, MST, k-cut)
- Meta-heuristics (genetic algorithms, simulated annealing)
- Exact models

Cell Formation Problem

Grouping efficacy suggested by Kumar and Chandrasekharan (1990)

$$\tau = \frac{n_1 - n_1^{out}}{n_1 + n_0^{in}} = \frac{n_1^{in}}{n_1 + n_0^{in}}$$

n_1 – a number of ones in the machine-part matrix

n_1^{in} – a number of ones inside the cells

n_0^{in} – a number of zeros inside the cells

Suggested methods

- Multi-start local search algorithm (MSLSA)
- 3 index BLP model
- 2 index BLP model

MSLSA

2 stages

Generate different random cells configurations

Apply local search procedure for each configuration

Stage 1

Generate different random cells configurations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
2	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
3	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0
4	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
5	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
6	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
7	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
8	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
9	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
10	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
11	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
12	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
13	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
15	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
2	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
3	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0
4	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
5	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
6	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
7	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
8	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
9	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
10	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
11	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
12	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
13	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
15	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
2	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
3	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0
4	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
5	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
6	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
7	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
8	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
9	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
10	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
11	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
12	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0
13	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
15	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0

MSLSA

Stage 2

Apply local search procedure for each configuration

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	0	0	0	0	0	0	0
2	1	0	1	1	1	1	1	0	0	1	0	0
3	0	0	1	1	1	1	1	1	1	0	0	0
4	0	0	0	0	0	1	1	1	1	1	0	0
5	0	0	0	0	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	1	1	1	0	1	0
7	0	0	0	0	0	0	0	0	0	0	1	1
8	0	0	0	0	0	0	0	0	0	0	1	1

Before move (efficacy 50%)

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	0	0	0	0	0	0	0
2	1	0	1	1	1	1	1	0	0	1	0	0
3	0	0	1	1	1	1	1	1	1	0	0	0
4	0	0	0	0	0	1	1	1	1	1	0	0
5	0	0	0	0	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	1	1	1	0	1	0
7	0	0	0	0	0	0	0	0	0	0	1	1
8	0	0	0	0	0	0	0	0	0	0	1	1

After move (efficacy 62.79%)

Computational results

№	Source	M	P	Efficacy,%		
				ACO	SA	MSLSA
1	King and Nakornchai (1982)	5	7	82.35	82.35	**82.35**
2	Waghodekar and Sahu (1984)	5	7	69.57	69.57	**69.57**
3	Seifoddini (1989)	5	18	79.59	79.59	**79.59**
4	Kusiak (1992)	6	8	76.92	76.92	**76.92**
5	Kusiak and Chow (1987)	7	11	60.87	60.87	**60.87**
6	Boctor (1991)	7	11	70.83	70.83	**70.83**
7	Seifoddini and Wolfe (1986)	8	12	69.44	69.44	**69.44**
8	Chandrasekharan and Rajagopalan (1989a,b)	8	20	85.25	85.25	**85.25**
9	Chandrasekharan and Rajagopalan (1989a,b)	8	20	58.72	58.72	**58.72**
10	Mosier and Taube (1985a)	10	10	75	75	**75**
11	Chan and Milner (1982)	10	15	92	92	**92**
12	Askin and Subramanian (1987)	14	24	72.06	72.06	**72.06**
13	Stanfel (1985)	14	24	71.83	71.83	**71.83**
14	McCormick et al. (1972)	16	24	52.75	53.26	**53.26**
15	Srinivasan et al. (1990)	16	30	68.99	68.99	**68.99**
16	King (1980)	16	43	57.53	57.53	**57.53**
17	Carrie (1973)	18	24	57.73	57.73	**57.73**
18	Mosier and Taube (1985b)	20	20	43.45	43.45	**43.45**

Computational results

№	Source	M	P	Efficacy,%		
				ACO	SA	MSLSA
19	Kumar et al. (1986)	20	23	50.81	50.81	50.81
20	Carrie (1973)	20	35	77.91	77.91	77.91
21	Boe and Cheng (1991)	20	35	57.98	57.98	57.98
22	Chandrasekharan and Rajagopalan (1989a,b)	24	40	100	100	100
23	Chandrasekharan and Rajagopalan (1989a,b)	24	40	85.11	85.11	85.11
24	Chandrasekharan and Rajagopalan (1989a,b)	24	40	73.51	73.51	73.51
25	Chandrasekharan and Rajagopalan (1989a,b)	24	40	53.29	53.29	53.29
26	Chandrasekharan and Rajagopalan (1989a,b)	24	40	48.95	48.95	48.95
27	Chandrasekharan and Rajagopalan (1989a,b)	24	40	47.26	47.26	46.26
28	McCormick et al. (1972)	27	27	54.82	54.82	54.82
29	Carrie (1973)	28	46	47.08	47.23	47.23
30	Kumar and Vannelli (1987)	30	41	63.31	63.31	63.31
31	Stanfel (1985)	30	50	59.77	59.77	59.77
32	Stanfel (1985)	30	50	50.83	50.83	50.83
33	King and Nakornchai (1982)	30	90	47.11	47.14	48.01
34	McCormick et al. (1972)	37	53	60.64	60.64	60.64
35	Chandrasekharan and Rajagopalan (1989a,b)	40	100	84.03	84.03	84.03

Summary

- 33 of 35 instances - greater or equal
- 1 of 35 – better result
- 1 of 35 – worse result

A new competitive approach

Possible enhancements:

- Deterministic enumeration instead of random choices

EXACT MODELS

Elbenani & Ferland (2012)

"Cell Formation Problem Solved Exactly with the Dinkelbach Algorithm"

Restricted formulation – fixed number of production cells

Problem

Work out a mathematical model that provides global optimal solution for the CFP (with variable number cells)

Key ideas

1. Find upper bound for number of zeroes inside the optimal solution based on current best result and total number of ones

$$\tau \leq \tau^* = \frac{n_1^{in}}{n_1 + n_0^{in}} \qquad n_0^{in} \leq \left[\frac{1-\tau^*}{\tau^*} n_1\right]$$

2. Divide problem into separated sub problems with number of zeroes inside from 0 to the upper bound

Table 8: Solutions for all possible values of n_0^{in}

#	Size	Zeroes Inside								
		0	1	2	3	4	5	6	7	8
1	5x7	78.57	80.00	81.25	**82.35**					
2	5x7	60.00	61.90	63.64	**69.57**	62.50	68.00	63.54	55.56	60.71

Variables

$$a_{ij} = \begin{cases} 1, & \text{if part } j \text{ is processed by machine } i \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ik} = \begin{cases} 1, & \text{if machine } i \text{ is assigned to cell } k \\ 0, & \text{otherwise} \end{cases}$$

Variables

$$y_{jk} = \begin{cases} 1, & \text{if part } j \text{ is assigned to cell } k \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ijk} = \begin{cases} 1, & \text{if both machine } i \text{ and part } j \text{ are assigned to cell } k \\ 0, & \text{otherwise} \end{cases}$$

Objective

Maximizing number of ones inside the solution

$$max \sum_{i=1}^{m} \sum_{j=1}^{p} \sum_{k=1}^{c} a_{ij} z_{ijk}$$

Constraints

Product linearization constraints

$$z_{ijk} \leq x_{ik} \quad \forall i = 1, \ldots, m, \ \forall j = 1, \ldots, p, \ \forall k = 1, \ldots, c$$

$$z_{ijk} \leq y_{jk} \quad \forall i = 1, \ldots, m, \ \forall j = 1, \ldots, p, \ \forall k = 1, \ldots, c$$

$$z_{ijk} \geq x_{ik} + y_{jk} - 1 \quad \forall i = 1, \ldots, m, \ \forall j = 1, \ldots, p, \ \forall k = 1, \ldots, c$$

Constraints

Machine/part assignment constraints

$$\sum_{k=1}^{c} x_{ik} = 1 \quad \forall i = 1, \ldots, m$$

$$\sum_{k=1}^{c} y_{jk} = 1 \quad \forall j = 1, \ldots, p$$

Constraints

Prohibiting machines without parts and parts without machines

$$\sum_{i=1}^{m}\sum_{j=1}^{p} z_{ijk} \geq \sum_{i=1}^{m} x_{ik} \quad \forall k = 1, \ldots, c$$

$$\sum_{i=1}^{m}\sum_{j=1}^{p} z_{ijk} \geq \sum_{j=1}^{p} y_{jk} \quad \forall k = 1, \ldots, c$$

Constraints

Fixing number of zeroes inside the solution

$$\sum_{i=1}^{m}\sum_{j=1}^{p}\sum_{k=1}^{c}(1-a_{ij})z_{ijk}=n_0^{in}$$

Summary

- 13 of 35 test instances – global optimal solutions found
- A huge number of variables and constraints (slows solving process)

Variables

$$a_{ij} = \begin{cases} 1, & \text{if part } j \text{ is processed by machine } i \\ 0, & \text{otherwise} \end{cases}$$

$$m_{ij} = \begin{cases} 1, & \text{if machine } i \text{ is in the same production cell with machine } j \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ik} = \begin{cases} 1, & \text{if machine } i \text{ is in the same production cell with part } k \\ 0, & \text{otherwise} \end{cases}$$

Objective

Maximizing number of ones inside the solution

$$max \sum_{i=1}^{m} \sum_{k=1}^{p} a_{ik} z_{ik}$$

Constraints

Assignment constraints

$$2m_{ij} - z_{ik} - z_{jk} \geq -1 \quad \forall i = 1, \ldots, m,\ \forall j = i, \ldots, m,\ \forall k = 1, \ldots, p$$

$$z_{ik} - z_{jk} - m_{ij} \geq -1.5 \quad \forall i = 1, \ldots, m,\ \forall j = i, \ldots, m,\ \forall k = 1, \ldots, p$$

$$z_{jk} - z_{ik} - m_{ij} \geq -1.5 \quad \forall i = 1, \ldots, m,\ \forall j = i, \ldots, m,\ \forall k = 1, \ldots, p$$

Constraints

Prohibiting machines without parts and parts without machines

$$\sum_{k=1}^{p} z_{ik} \geq 1 \quad \forall i = 1, \ldots, m$$

$$\sum_{i=1}^{m} z_{ik} \geq 1 \quad \forall k = 1, \ldots, p$$

Constraints

Fixing number of zeroes inside the solution

$$\sum_{i=1}^{m}\sum_{k=1}^{p}(1-a_{ik})z_{ik} = n_0^{in}$$

Results

#	Source	M	P	Efficiency value,%			Time, sec		
				Elbenani & Ferland	3 index	2 index	Elbenani & Ferland	3 index	2 index
1	King [15]	5	7	82.35	82.35	82.35	2.3	0.63	0.11
2	Waghodekar & Sahu [36]	5	7	69.57	69.57	69.57	1.6	2.29	2.28
3	Seifoddini [43]	5	18	79.59	79.59	79.59	3.1	5.69	1.59
4	Kusiak [20]	6	8	76.92	76.92	76.92	2.0	1.86	1,06
5	Kusiak & Chow [41]	7	11	60.87	60.87	60.87	30.6	9.14	1.38
6	Boctor [42]	7	11	70.83	70.83	70.83	4.3	5.15	0.5
7	Seifoddini & Wolfe [32]	8	12	69.44	69.44	69.44	9.6	13.37	1.35
8	Chandrasekharan & Rajagopalan [8]	8	20	85.25	85.25	85.25	3.1	18.33	5.29
9	Chandrasekharan & Rajagopalan [9]	8	20	58.72	58.72	58.72	3.5	208.36	51.89
10	Mosier & Taube [44]	10	10	75	75	75	1.1	6.25	0.55
11	Chan & Milner [7]	10	15	92.00	92.00	92.00	1.63	2.93	0.49
12	Askin & Subramanian [45]	14	24	72.06	72.06	72.06	2188.7	259.19	5.11
13	Stanfel [35]	14	24	71.83	71.83	71.83	593.2	179.21	7.6
14	McCormick et al. [24]	16	24	53.26	51.61*	53.26	15130.5	*	972
15	Kumar et al. [47]	16	30	69.53	68.99*	68.99	252	*	353.45
16	King [15]	16	43	57.53	57.53*	57.53	183232.5	*	2375.07
17	Carrie [6]	18	24	57.73	57.73*	57.73	2345.6	*	381.12
18	Mosier & Taube [28]	20	20	43.06*	38.71*	43.45	*	*	977064.99
19	Kumar et al. [47]	20	23	50.81	46.72*	50.81	131357.5	*	21029.5
20	Carrie [6]	20	35	77.91	77.85*	77.91	31.1	*	250.54
21	Boe & Cheng [3]	20	35	57.98	46.75*	57.98	14583	*	22855.33
22	Chandrasekharan & Rajagopalan [11]	24	40	100	100	100	11.3	1.64	0.03
23	Chandrasekharan & Rajagopalan [11]	24	40	85.11	85.11*	85.11	230.7	*	772.29
24	Chandrasekharan & Rajagopalan [11]	24	40	73.51	56.49*	73.51	1101.1	*	5448
25	Chandrasekharan & Rajagopalan [11]	24	40	53.29	43.56*	53.29	*	*	29306

Summary

- **25 of 35 test instances – global optimal solutions found**
- **17 of 25 instances – faster than two others (up to 437 times faster)**

Thanks for your attention !