Power and shift independent imaging of coherent sources

Dmitrii Altukhov, Alex Ossadtchii

MEG-Centre, MSUPE
HSE, computer science department

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1. Motivation
2. Algorithm
3. Simulations
4. Real data results

ECoG

invasive

fMRI

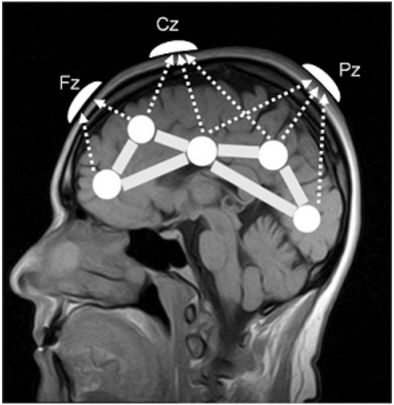
bad temporal resolution ($\approx$ 1s)

MEG

EEG

EEG and MEG allow to measure electromagnetic brain activity directly, noninvasively and with good temporal resolution

Two problems: signal leakage and ill-posedness



Common approach today

1. Compute inverse (ROI, atlas or sparce grid)
2. Select seeds or go all-to-all
3. Use measures insensitive to signal leakage, e.g:
    - ImCoh [Nolte et al., 2004]
    - PLI
    - wPLI
    - Geometric correction scheme [Wens et al., Human Brain Mapping, 2015]

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Problems of this approach

1. SL-proof connectivity measures lead to SNR loss
2. Inverse solution is affected by signal leakage

The idea of PSIICOS on a toy problem

Generative model

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}(t) = \begin{pmatrix} g_1^1 & g_2^1 \\ g_1^2 & g_2^1 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}(t) =$$

$$= \begin{pmatrix} g_1^1 \\ g_1^2 \end{pmatrix} s_1(t) + \begin{pmatrix} g_2^1 \\ g_2^2 \end{pmatrix} s_2(t) = \vec{g}_1 s_1(t) + \vec{g_2} s_2(t) \quad (1)$$

$m_{1,2}(t)$ - MEG/EEG measurements
$\{g_i^j\}$ - matrix of a forward model
$s_{1,2}(t)$ - unknown timeseries on cortex

Time-frequency transformation

Apply time-frequency transform to (1)...

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix}(f,t) = \vec{g}_1 S_1(f,t) + \vec{g_2} S_2(f,t) \tag{2}$$

... and write cross-spectrum:

$$\mathbf{C}^{MM}(t,f) \overset{def}{=} \mathbf{E}\{\mathbf{M}(t,f)\mathbf{M}^H(t,f)\} \tag{3}$$

N.B.

M_1, M_2, S_1, S_2 after time-frequency transformation are complex

Let's substitute (2) into (3)

$$\mathbf{C}^{MM}(t,f) = \mathbf{E}\left\{ \begin{pmatrix} M_1\bar{M}_1 & M_1\bar{M}_2 \\ M_2\bar{M}_1 & M_2\bar{M}_2 \end{pmatrix}(f,t) \right\} =$$

$$\mathbf{E}\left\{ \Big(\vec{g}_1 S_1(f,t) + \vec{g_2} S_2(f,t)\Big) \cdot \Big(\vec{g}_1^T \bar{S}_1(f,t) + \vec{g_2}^T \bar{S}_2(f,t)\Big) \right\} \quad (4)$$

Let's substitute (2) into (3)

$$\mathbf{C}^{MM}(t,f) = \mathbf{E}\left\{ \begin{pmatrix} M_1\bar{M}_1 & M_1\bar{M}_2 \\ M_2\bar{M}_1 & M_2\bar{M}_2 \end{pmatrix}(f,t) \right\} =$$

$$\mathbf{E}\left\{ \left(\vec{g}_1 S_1(f,t) + \vec{g}_2 S_2(f,t)\right) \cdot \left(\vec{g}_1^T \bar{S}_1(f,t) + \vec{g}_2{}^T \bar{S}_2(f,t)\right) \right\} \quad (4)$$

N.B.

$\vec{g}_i$ are real $\implies \vec{g}_i{}^H = \vec{g}_i{}^T$

Let's substitute (2) into (3)

$$\mathbf{C}^{MM}(t,f) = \mathbf{E}\left\{ \begin{pmatrix} M_1\bar{M}_1 & M_1\bar{M}_2 \\ M_2\bar{M}_1 & M_2\bar{M}_2 \end{pmatrix} (f,t) \right\} =$$

$$\mathbf{E}\left\{ \Big(\vec{g_1}S_1(f,t) + \vec{g_2}S_2(f,t)\Big) \cdot \Big(\vec{g_1}^T\bar{S}_1(f,t) + \vec{g_2}^T\bar{S}_2(f,t)\Big) \right\}$$
$$= \vec{g_1}\vec{g_1}^T\mathbf{E}\{S_1(f,t)\bar{S}_1(f,t)\} + \vec{g_1}\vec{g_2}^T\mathbf{E}\{S_1(f,t)\bar{S}_2(f,t)\} +$$
$$+ \vec{g_2}\vec{g_1}^T\mathbf{E}\{S_2(f,t)\bar{S}_1(f,t)\} + \vec{g_2}\vec{g_2}^T\mathbf{E}\{S_2(f,t)\bar{S}_2(f,t)\} \quad (4)$$

Let's substitute (2) into (3)

$$\mathbf{C}^{MM}(t,f) = \\ = \vec{g_1}\vec{g_1}^T\mathbf{E}\{S_1(f,t)\bar{S_1}(f,t)\} + \vec{g_1}\vec{g_2}^T\mathbf{E}\{S_1(f,t)\bar{S_2}(f,t)\} + \\ + \vec{g_2}\vec{g_1}^T\mathbf{E}\{S_2(f,t)\bar{S_1}(f,t)\} + \vec{g_2}\vec{g_2}^T\mathbf{E}\{S_2(f,t)\bar{S_2}(f,t)\} \quad (4)$$

Finally, we've got

$$\begin{pmatrix} c_{11}^{MM} & c_{12}^{MM} \\ c_{21}^{MM} & c_{22}^{MM} \end{pmatrix} = \\ = \vec{g_1}\vec{g_1}^T c_{11}^{SS} + \vec{g_1}\vec{g_2}^T c_{12}^{SS} + \vec{g_2}\vec{g_1}^T c_{21}^{SS} + \vec{g_2}\vec{g_2}^T c_{22}^{SS} \quad (5)$$

Or in matrix form:

$$\begin{pmatrix} c_{11}^{MM} & c_{12}^{MM} \\ c_{21}^{MM} & c_{22}^{MM} \end{pmatrix} =$$

$$= \begin{pmatrix} g_1^1 g_1^1 & g_1^1 g_1^2 \\ g_1^2 g_1^1 & g_1^2 g_1^2 \end{pmatrix} c_{11}^{SS} + \begin{pmatrix} g_1^1 g_2^1 & g_1^1 g_2^2 \\ g_1^2 g_2^1 & g_1^2 g_2^2 \end{pmatrix} c_{12}^{SS} +$$

$$+ \begin{pmatrix} g_2^1 g_1^1 & g_2^1 g_1^2 \\ g_2^2 g_1^1 & g_2^2 g_1^2 \end{pmatrix} c_{21}^{SS} + \begin{pmatrix} g_2^1 g_2^1 & g_2^1 g_2^2 \\ g_2^2 g_2^1 & g_2^2 g_2^2 \end{pmatrix} c_{22}^{SS} \tag{5}$$

Or in matrix form:

$$
\begin{aligned}
\begin{pmatrix} c_{11}^{MM} & c_{12}^{MM} \\ c_{21}^{MM} & c_{22}^{MM} \end{pmatrix} = \\
= \begin{pmatrix} g_1^1 g_1^1 & g_1^1 g_1^2 \\ g_1^2 g_1^1 & g_1^2 g_1^2 \end{pmatrix} c_{11}^{SS} + \begin{pmatrix} g_1^1 g_2^1 & g_1^1 g_2^2 \\ g_1^2 g_2^1 & g_1^2 g_2^2 \end{pmatrix} c_{12}^{SS} + \\
+ \begin{pmatrix} g_2^1 g_1^1 & g_2^1 g_1^2 \\ g_2^2 g_1^1 & g_2^2 g_1^2 \end{pmatrix} c_{21}^{SS} + \begin{pmatrix} g_2^1 g_2^1 & g_2^1 g_2^2 \\ g_2^2 g_2^1 & g_2^2 g_2^2 \end{pmatrix} c_{22}^{SS} \quad (5)
\end{aligned}
$$

Signal leakage

ImCoh [Nolte et al., 2004] : drop real part of eq. 5.

Or in matrix form:

$$\begin{pmatrix} c_{11}^{MM} & c_{12}^{MM} \\ c_{21}^{MM} & c_{22}^{MM} \end{pmatrix} = \\ = \begin{pmatrix} g_1^1 g_1^1 & g_1^1 g_1^2 \\ g_1^2 g_1^1 & g_1^2 g_1^2 \end{pmatrix} c_{11}^{SS} + \begin{pmatrix} g_1^1 g_2^1 & g_1^1 g_2^2 \\ g_1^2 g_2^1 & g_1^2 g_2^2 \end{pmatrix} c_{12}^{SS} + \\ + \begin{pmatrix} g_2^1 g_1^1 & g_2^1 g_1^2 \\ g_2^2 g_1^1 & g_2^2 g_1^2 \end{pmatrix} c_{21}^{SS} + \begin{pmatrix} g_2^1 g_2^1 & g_2^1 g_2^2 \\ g_2^2 g_2^1 & g_2^2 g_2^2 \end{pmatrix} c_{22}^{SS} \quad (5)$$

Signal leakage

ImCoh [Nolte et al., 2004] : drop real part of eq. 5.

Vectorized equation:

$$\begin{pmatrix} c_{11}^{MM} \\ c_{12}^{MM} \\ c_{21}^{MM} \\ c_{22}^{MM} \end{pmatrix} = \begin{pmatrix} g_1^1 g_1^1 \\ g_1^1 g_1^2 \\ g_1^2 g_1^1 \\ g_1^2 g_1^2 \end{pmatrix} c_{11}^{SS} + \begin{pmatrix} g_1^1 g_2^1 \\ g_1^1 g_2^2 \\ g_1^2 g_2^1 \\ g_1^2 g_2^2 \end{pmatrix} c_{12}^{SS} + \begin{pmatrix} g_2^1 g_1^1 \\ g_2^1 g_1^2 \\ g_2^2 g_1^1 \\ g_2^2 g_1^2 \end{pmatrix} c_{21}^{SS} + \begin{pmatrix} g_2^1 g_2^1 \\ g_2^1 g_2^2 \\ g_2^2 g_2^1 \\ g_2^2 g_2^2 \end{pmatrix} c_{22}^{SS}$$

Separating powers and interactions

$$
\begin{pmatrix} c_{11}^{MM} \\ c_{12}^{MM} \\ c_{21}^{MM} \\ c_{22}^{MM} \end{pmatrix} = \begin{pmatrix} g_1^1 g_1^1 \\ g_1^1 g_1^2 \\ g_1^2 g_1^1 \\ g_1^2 g_1^2 \end{pmatrix} c_{11}^{SS} + \begin{pmatrix} g_1^1 g_2^1 \\ g_1^1 g_2^2 \\ g_1^2 g_2^1 \\ g_1^2 g_2^2 \end{pmatrix} c_{12}^{SS} + \begin{pmatrix} g_2^1 g_1^1 \\ g_2^1 g_1^2 \\ g_2^2 g_1^1 \\ g_2^2 g_1^2 \end{pmatrix} c_{21}^{SS} + \begin{pmatrix} g_2^1 g_2^1 \\ g_2^1 g_2^2 \\ g_2^2 g_2^1 \\ g_2^2 g_2^2 \end{pmatrix} c_{22}^{SS}
$$

Project equation to the orthogonal complement of these vectors

$\mathbf{P}$:

$$
\begin{gathered}
\mathbf{F} = [vec(\mathbf{g_1 g_1^T}), vec(\mathbf{g_2 g_2^T})] \\
\mathbf{F} = \mathbf{USV^T} \\
\mathbf{U_2} = [\mathbf{u_1}, \mathbf{u_2}] \\
\mathbf{P} = \mathbf{I} - \mathbf{U_2 U_2^T} \\
\boldsymbol{C}^{\perp} = \boldsymbol{P} vec(\boldsymbol{C}^{MM})
\end{gathered}
$$

Finally we get

$$\mathbf{P} \cdot \begin{pmatrix} c_{11}^{MM} \\ c_{12}^{MM} \\ c_{21}^{MM} \\ c_{22}^{MM} \end{pmatrix} (t) = \mathbf{P} \cdot \begin{pmatrix} g_1^1 g_2^1 \\ g_1^1 g_2^2 \\ g_1^2 g_2^1 \\ g_1^2 g_2^2 \end{pmatrix} c_{12}^{SS}(t) + \mathbf{P} \cdot \begin{pmatrix} g_2^1 g_1^1 \\ g_2^1 g_1^2 \\ g_2^2 g_1^1 \\ g_2^2 g_1^2 \end{pmatrix} c_{21}^{SS}(t)$$

We know

$$\mathbf{P}, \begin{pmatrix} c_{11}^{MM} \\ c_{12}^{MM} \\ c_{21}^{MM} \\ c_{22}^{MM} \end{pmatrix} (t), \begin{pmatrix} g_1^1 g_2^1 \\ g_1^1 g_2^2 \\ g_1^2 g_2^1 \\ g_1^2 g_2^2 \end{pmatrix}, \begin{pmatrix} g_2^1 g_1^1 \\ g_2^1 g_1^2 \\ g_2^2 g_1^1 \\ g_2^2 g_1^2 \end{pmatrix}$$

We want to find
$c_{12}^{SS}(t), c_{21}^{SS}(t)$

We need to solve

$$vec(\mathbf{C}^{\perp}(t)) = \sum_{i=1}^{L}\sum_{j=1}^{L} vec(\mathbf{g_i}\mathbf{g_j^T})^{\perp} c_{ij}^{ss}(t) + vec(\mathbf{C^{NN}}(t)) \qquad (7)$$

$c_{ij}^{ss}(t)$ are the unknown timeseries on cortex which we are to recover

PSIICOS objection:

$$vec(\mathbf{C}^{\perp}(t)) = \sum_{i=1}^{L}\sum_{j=1}^{L} vec(\mathbf{g_i g_j^T})^{\perp} c_{ij}^{ss}(t) + vec(\mathbf{C^{NN}}(t)) \quad (8)$$

Define new variables:

Let $\mathbf{\Omega} = vec(\mathbf{C}^{\perp}(t))$,
$\mathbf{\Gamma_k} = vec(\mathbf{g_i g_j^T})^{\perp}$, $\sigma_{\mathbf{k}}(t) = c_{ij}^{ss}$, $\mathbf{N}(t) = \mathbf{C^{NN}}(t)$; then (8) will look like:

$$\mathbf{\Omega}(t) = \sum_{k=1}^{L^2} \mathbf{\Gamma_k} \sigma_{\mathbf{k}}(t) + \mathbf{N}(t) \quad (9)$$

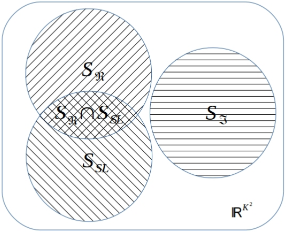
One way to estimate coherent sources is to look at correlation of topographies with signal subspase

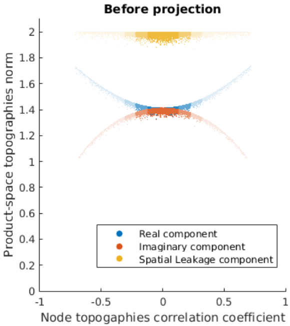
$$\mathbf{\Omega} = \mathbf{USV^T}$$

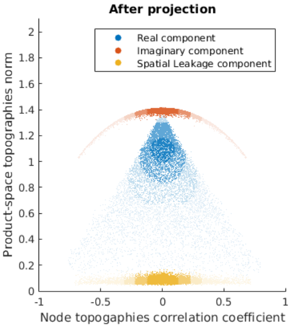
$$\mathbf{C_r} = [\mathbf{u_1}, \mathbf{u_2}, ..., \mathbf{u_r}]$$

$$A^{(n)} = \left\{ \sigma \;\middle|\; \left\| \mathbf{\Gamma}_\sigma^\mathbf{T} \mathbf{C_r} \right\| > threshold \right\}$$

Projection effect

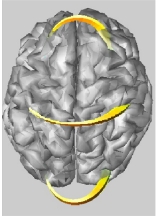




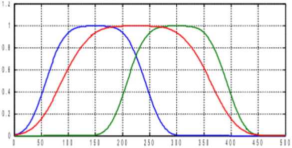


- Three phase-locked networks
- $1/f$ brain noise
- Networks activity overlap in time
- Induced activity (same frequencies = 10 Hz, different envelopes for each network)
- Bandpass filter 2-20 Hz
- Constant phase shifts $(\pi/20, \pi/2 - \pi/20)$ plus random phase
- Solved on grid with 1503 verticies

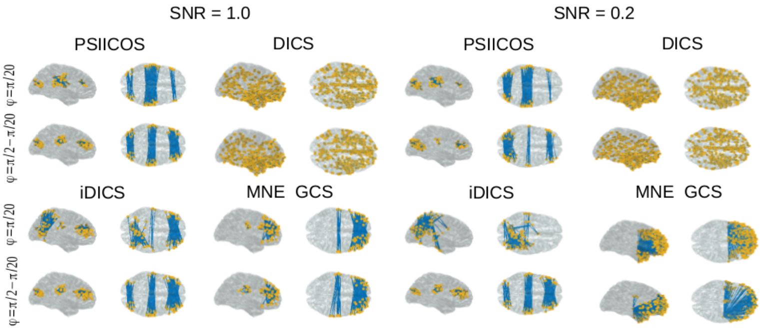
Source locations:



Synchrony profiles:



Comparison to other algorithms on simulated data



Precision-recall curves for simulations

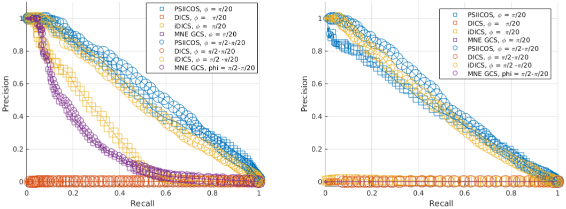


Figure: Three networks detection (left: SNR=1, right: SNR=0.2)

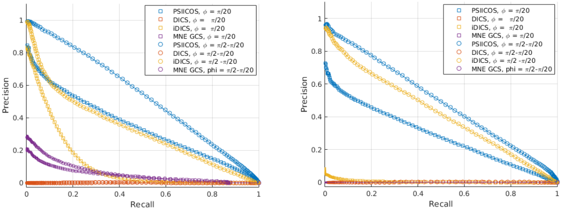
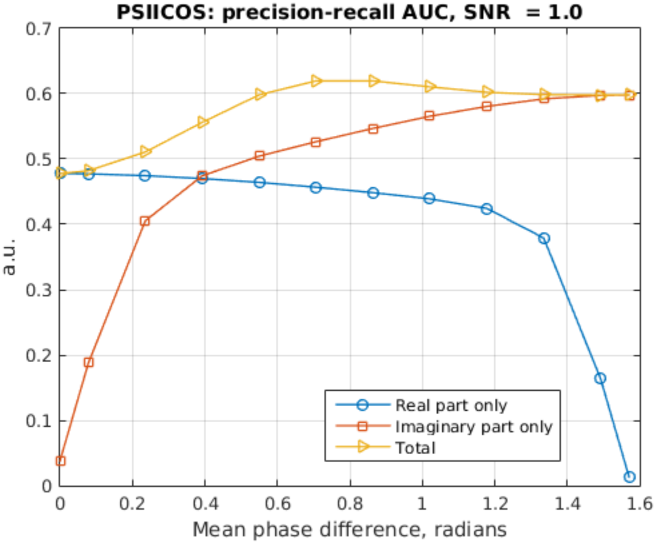
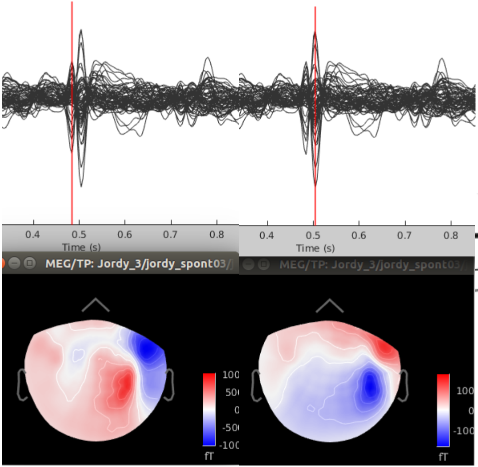
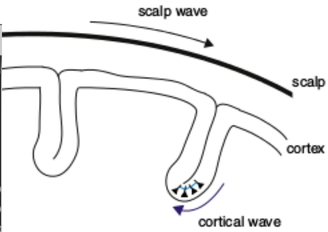


Figure: Monte-Carlo networks simulation (left: SNR=1, right: SNR=0.2)

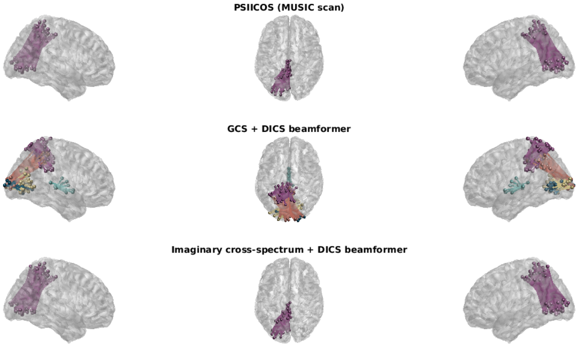
Performance for different phase shifts




0.4
0.5
0.6
0.7
0.8
Time (s)
MEG/TP: Jordy_3/jordy_spont03/
100
50
0
-50
-100
fT
0.4
0.5
0.6
0.7
0.8
Time (s)
MEG/TP: Jordy_3/jordy_spont03/
100
0
-100
fT

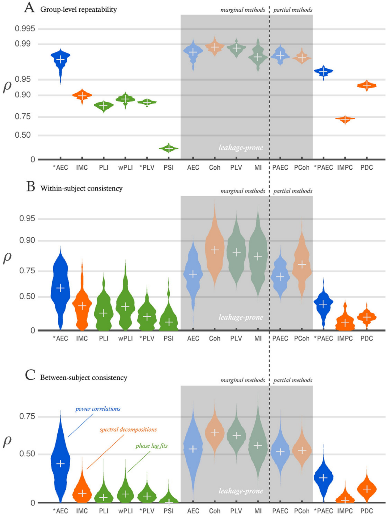

scalp wave
scalp
cortex
cortical wave

Rotation speed: 40 rotations per second. Initial phase is random

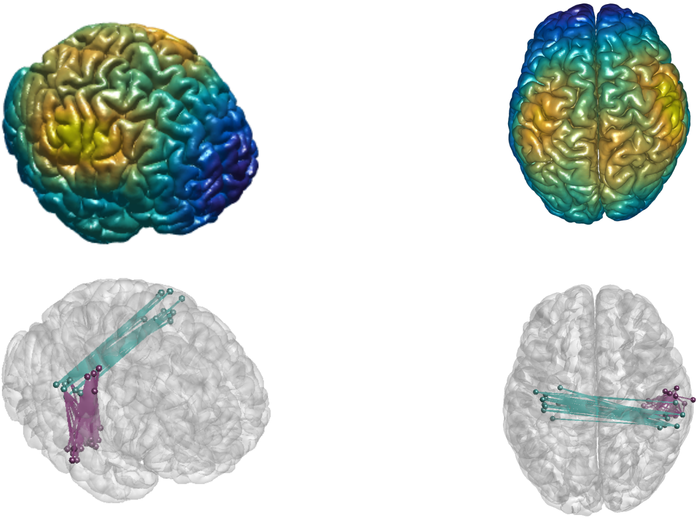


Connectivity measures consistency

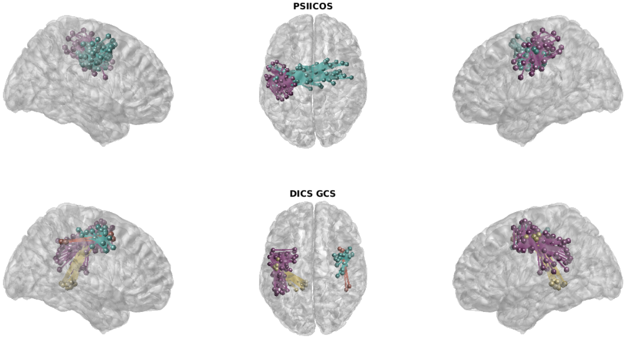
How reliable connectivity measures are? [Colclough et al., Neuroimage, 2015]



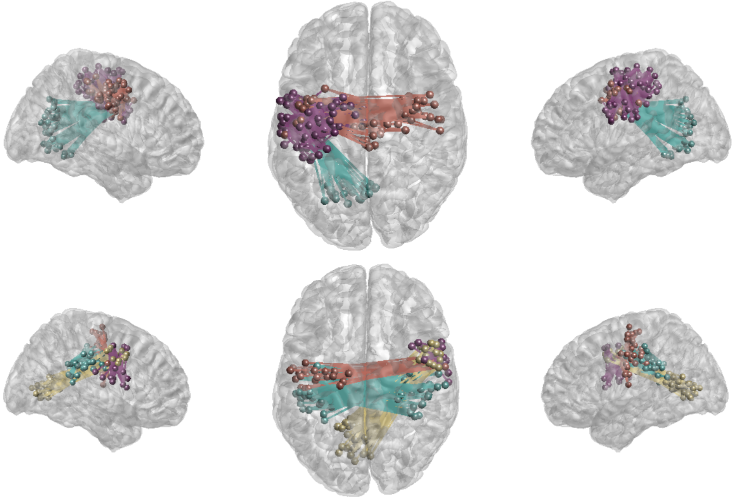
Band: 8-12 Hz, stimulus: audio odd-ball, trial: 0.4 - 0.7 s



300 bootstrap iterations, band: 16-25 Hz



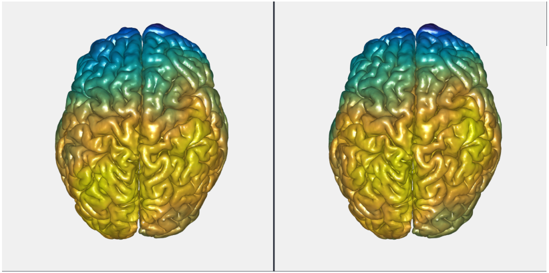
Band: 16-25, 200 bootstrap iterations

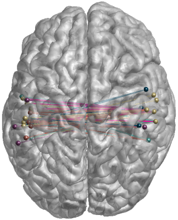


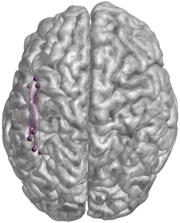
Monte-Carlo procedure Avarvand et al., IFMBE proceedings, 2010

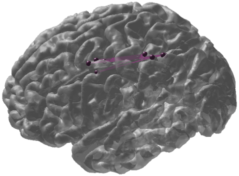
1. Compute ICA on real data
2. shift ICA components randomly for each trial
3. project back to sensors
4. repeat N times
5. compute maximum subspace correlation on each iteration
6. see, how real data max subspace correlation fits in obtained distribution

Surrogate data results





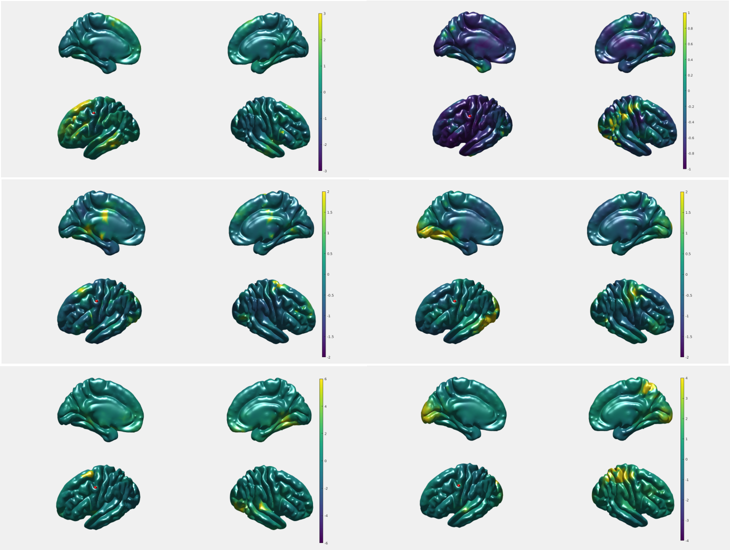




$p < 0.01$

Real projected vs Imag

Voluntarily finger movement, 3 subjects, real projected cross-spectrum (left) vs imaginary cross-spectrum



Thank you!