Алгоритмы кластеризации в моделях случайных графов

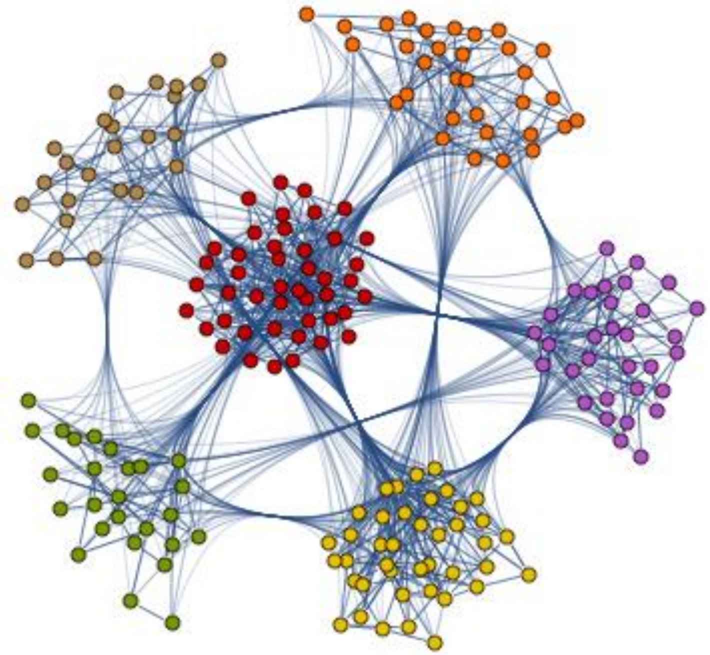
Аспирант второго года обучения Казаков М.А.

Утвержденная тема диссертации: Алгоритмы кластеризации в моделях случайных графов

Научный руководитель: проф., д.ф.-м.н. Калягин В.А.

Задача кластеризации на графах

Разделить исходный граф на достаточно плотные подграфы, имеющие относительно малое количество связей между собой.



Задача кластеризации на графах

Вариации:

- Веса ребер
- Ориентация ребер
- Атрибуты вершин
- Динамические сети

- Пересекающиеся сообщества
- Нечеткая кластеризация

Алгоритмы

- Divisive algorithms
- Modularity-based methods
- Spectral algorithms
- Dynamic algorithms
- Methods based on statistical inference
- …

Fortunato, Santo. "Community detection in graphs." Physics reports 486.3 (2010): 75-174.

Benchmarks

- Real-life
- Generated

Benchmarks: generated

- Stochastic block model

- Degree-corrected block model

Karrer, Brian, and Mark EJ Newman. "Stochastic blockmodels and community structure in networks." Physical Review E 83.1 (2011): 016107.

- LFR

Lancichinetti, Andrea, Santo Fortunato, and Filippo Radicchi. "Benchmark graphs for testing community detection algorithms." Physical review E 78.4 (2008): 046110.

Stochastic block model

- The number *n* of vertices
- A partition of the vertex set V = {1,…,n} into disjoint subsets $C_1,\dots,C_r$, called *communities*
- A symmetric *r×r* matrix *P* of inter-community edge probabilities

$$P_{ij} = Pr\{edge(u,v)\},\ u \in C_i \text{ and } v \in C_j$$

Erdős–Rényi model

- $|V| = n$ vertices
- $r = 1$ communities
- $P = p$

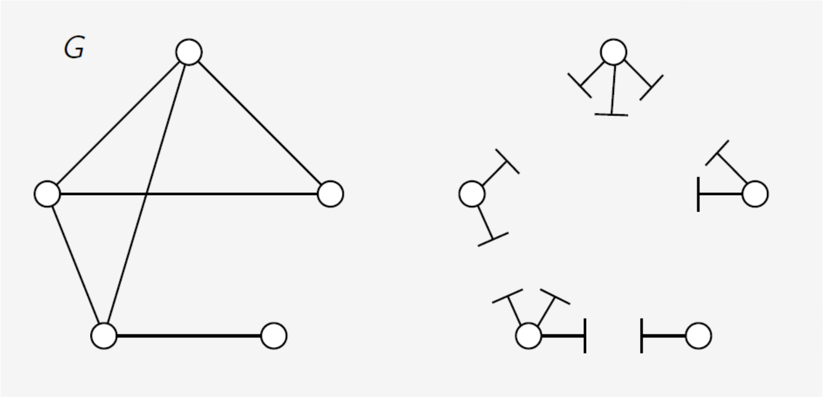
Planted partition model

- $|V| = n$ vertices, $r = 2$ communities
- $|C_1| = |C_2| = n/2$
- $P = \begin{pmatrix} p_{in} & p_{out} \\ p_{out} & p_{in} \end{pmatrix}$

Modularity

$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - P_{ij})\, \delta(C_i, C_j)$$

Null model:



$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(C_i, C_j)$$

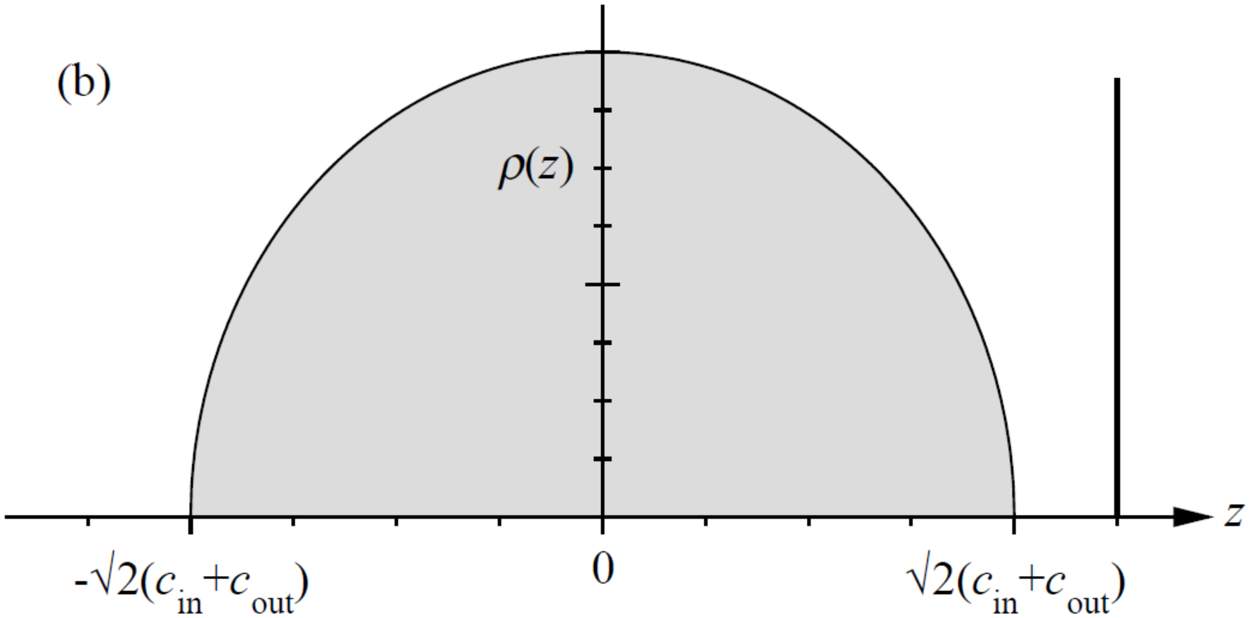
Detectability of community structure

- A – adjacency matrix $n \times n$, where $A_{ij} = 1$ if there is an edge(i, j) and 0 otherwise
- $\langle A \rangle = \left(\begin{array}{c|c} p_{in} & p_{out} \\ \hline p_{out} & p_{in} \end{array}\right)$
- $\langle A \rangle = \frac{1}{2}(c_{in} + c_{out})11^T + \frac{1}{2}(c_{in} - c_{out})uu^T$, $c_{in} = np_{in}$, $c_{out} = np_{out}$

$$\mathbf{1} = (\underbrace{1,1,1,\ldots}_{n})/\sqrt{n},\ u = (\underbrace{1,1,\ldots,1}_{n/2},\underbrace{-1,-1,\ldots,-1}_{n/2})/\sqrt{n}$$

$$\mathbf{B} = \frac{1}{2}(c_{\text{in}} - c_{\text{out}})\,\mathbf{u}\mathbf{u}^T + \mathbf{X} = \mathbf{A} - \frac{1}{2}(c_{\text{in}} + c_{\text{out}})\,\mathbf{1}\mathbf{1}^T$$

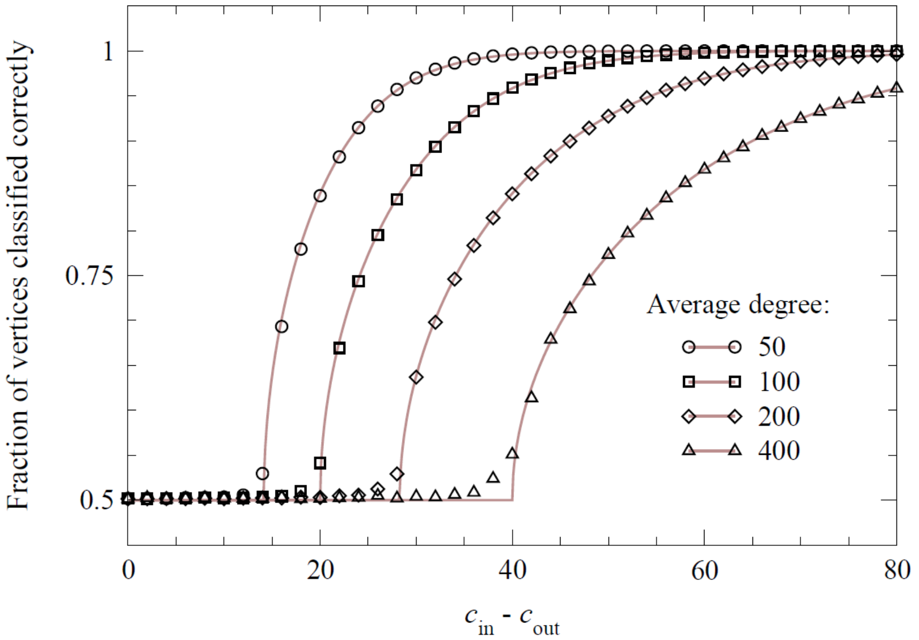
Detectability of community structure



$$c_{\text{in}} - c_{\text{out}} = \sqrt{2(c_{\text{in}} + c_{\text{out}})}$$

$$c_{\text{in}} - c_{\text{out}} = \sqrt{q[c_{\text{in}} + (q-1)c_{\text{out}}]}$$

Detectability of community structure



$$\frac{1}{2}\left[1 + \operatorname{erf}\sqrt{\alpha^2/2(1-\alpha^2)}\right]$$

$$\alpha^2 = \frac{(c_{\text{in}} - c_{\text{out}})^2 - 2(c_{\text{in}} + c_{\text{out}})}{(c_{\text{in}} - c_{\text{out}})^2}$$, $\operatorname{erf} x$ – Gaussian error function

Detectability of community structure

Nadakuditi, Raj Rao, and Mark EJ Newman. "Graph spectra and the detectability of community structure in networks." Physical review letters 108.18 (2012): 188701.

Decelle, Aurelien, et al. "Inference and phase transitions in the detection of modules in sparse networks." Physical Review Letters 107.6 (2011): 065701.

Задачи

Другие алгоритмы:

- Min Cut
- Normalized Cut

Другие модели:

- Degree-corrected block model
- LFR
- ERGMs

НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

Спасибо за внимание!