Syllabus for the course "Mathematical Statistics"

Area of Specialisation: 01.06.01 Mathematics and Mechanics

Doctoral programs in
01.01.03 Mathematical Physics
01.01.04 Geometry and Topology
01.01.05 Probability Theory and Mathematical Statistics
01.01.06 Mathematical Logics, Algebra and Theory of Numbers

Approved by the Academic Council of the Doctoral School in Mathematics
on 25th October, 2016

Moscow - 2016

Syllabus

1. Course Description
    a. Title of a Course: Mathematical Statistics
    b. Pre-requisites: the most basic part of probability theory, like distributions of random variables, mathematical expectations and variances for random variables, statement of the central limit theory (knowledge of the proof is not required).
    c. Course Type: optional
    d. Abstract: The main goal of mathematical statistics is an adaptation of the theoretical probabilistic models to some practical problems in economics, physics, medicine, social sciences. Typically the precise distribution or random process that describes some phenomenon is not known; however, some information can be extracted from the series of observations or repeated experiments; this data is be used to select the most appropriate model.
2. Learning Objectives
   To give a broad perspective on the subject and to work out the simplest examples.
3. Learning Outcomes
   Students will master fundamental concepts of mathematical statistics, will be able to apply them to problems in mathematics, economics, physics, etc.
4. Course Plan
   ∘ Statistical models, samples, descriptive statistics. Empirical approach: empirical distribution and Glivenko – Cantelli theorem.
   ∘ Parametric statistics: estimations and their main properties. Unbiased estimators. Efficient estimators. Cramer – Rao low bound. Consistent estimators. Fisher – Neumann factorization theorem. Rao – Blackwell theorem. Confidence intervals.
   ∘ Statistical hypothesis testing. Common test statistics. Null hypothesis statistical significance testing. Neumann–Pearson lemma and the most powerful test at the given significance level.
5. Reading List
    a. Required
    Hogg, R. V.
    Introduction to mathematical statistics / R. V. Hogg, J. McKean, A. T. Craig. – 7th ed. – Edinburgh: Pearson, 2014. – 650 с. – На англ. яз. - ISBN 978-1-292-02499-8.

b. Optional

Newbold, P.

Statistics for business and economics / P. Newbold, W. L. Carlson, B. Thorne. – Upper Saddle River: Pearson Education International, 2007. – 984 с. – На англ. яз. - ISBN 0-13-814250-5.

6. Grading System

Final grade = 0.5(Homeworks)+0.5(Final Exam).

The final grade is rounded to the nearest integer; half-integers are rounded upwards.

7. Guidelines for Knowledge Assessment

1. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \Gamma(\alpha, \beta)$. Find a pair of complete and sufficient statistics for this model.
2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Poisson distribution with parameter λ.
   (a) Find the UMVUE for λ.
   (b) Find the UMVUE for $\tau(\lambda) = e^{-\lambda}$. (Hint: Rao-Blackwell!)
3. Let $X_1, X_2, \ldots, X_n$ be a random sample Beta distribution with parameters $a = \theta$ and $b = 1$.
   (a) Find the UMVUE for $1/\theta$.
   (b) Find the UMVUE for $(\theta/(\theta+1))^n$.

8. Methods of Instruction: lectures and tutorials conducted by course instructors
9. Special Equipment and Software Support (if required)

   No requirements

Competences to be developed: UK-1, 2, 5, PK-1, OPK-1, 2 (according to *01.06.01 Mathematics and Mechanics* Educational Standard).