Syllabus for the course “Lie groups and algebras”

Area of Specialisation: 01.06.01 Mathematics and Mechanics
Doctoral programs in

01.01.02 Differential Equations, Dynamic Systems and Optimal Management
01.01.03 Mathematical Physics
01.01.04 Geometry and Topology
01.01.05 Probability Theory and Mathematical Statistics
01.01.06 Mathematical Logics, Algebra and Theory of Numbers

Approved by the Academic Council of the Doctoral School in Mathematics
on 24th October, 2017

Moscow - 2017

National Research University Higher School of Economics
Syllabus for the course “Lie groups and algebras” for PhD Programs, Area of Specialisation: 01.06.01 Mathematics and Mechanics

Syllabus

1. Course Description
    a. Title of a Course: Lie groups and algebras
    b. Pre-requisites: standard course of linear algebra.
    c. Course Type: optional
    d. Abstract

    We shall begin with the basics of the theory of Lie groups and Lie algebras. Then we shall provide an accessible introduction to the theory of finite-dimensional representations of classical groups on the example of the unitary groups U(N).
2. Learning Objectives

    To give a broad introduction to methods of Lie theory applicable to a wide range of problems in mathematics and science.
3. Learning Outcomes

    Participants will master basics of representation theory, and be able to perform particular computations with finite-dimensional representations of classical groups
4. Course Plan

∘ linear Lie groups and their Lie algebras
∘ universal enveloping algebras
∘ the Haar measure on a linear Lie group
∘ general facts about representations of compact groups and their characters
∘ radial part of Haar measure
∘ Weyl’s formula for characters of the unitary groups
∘ Weyl’s unitary trick
∘ classification and realization of representations.

5. Reading List
    a. Required

    A. Kirillov, Jr., Introduction to Lie groups and Lie algebras. CUP, 2008;
    https://www.math.stonybrook.edu/~kirillov/mat552/liegroups.pdf
    b. Optional

    W. Fulton, J. Harris, Representation theory. Springer 1991;
    Russian translation: 2017: Фултон, У.

Теория представлений: начальный курс / У. Фултон, Дж. Харрис; Пер. с англ. Е. Ю. Смирнова, Е. В. Шаройко; Под ред. С. М. Львовского. – М.: МЦНМО, 2017. – 583 с. - ISBN 9785443925455: 468.00.

6. Grading System

50% final exam, 30% written tests/midterms, 20% quizzes. The final grade is rounded up to the nearest integer (e.g., 9.1 rounds to 10).

7. Guidelines for Knowledge Assessment

Sample problems:

(1) Denote by $Spin(n)$ the universal cover of $SO_n(\mathbb{C})$. Prove the *accidental* isomorphisms between reductive groups

(a) $Spin(3)$ and $SL2(\mathbb{C})$; (b) $Spin(4)$ and $SL2(\mathbb{C}) \times SL2(\mathbb{C})$;

(c) $Spin(5)$ and $Sp4(\mathbb{C})$; (d) $Spin(6)$ and $SL4(\mathbb{C})$.

(2) Let V be the tautological representation of $G = SL_3(\mathbb{C})$. Decompose the tensor product of V and V^* into irreducible representations of G.

(3) Find the dimension of the irreducible representation of $GL_n(C)$ with the highest weight $2\backslash rho$.

8. Methods of Instruction: lectures, tutorials, self study
9. Special Equipment and Software Support (if required)

No requirements

Competences to be developed: UK-1, 2, 5, PK-1, OPK-1, 2 (according to *01.06.01 Mathematics and Mechanics* Educational Standard).